ECON120A: Confidence Intervals

Week 9

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WHAT IS A CONFIDENCE INTERVAL?

- In statistics, we rarely know the true **population parameter** (for example, the population mean μ).
- We collect a sample ($\{x_1, x_2, \ldots, x_n\}$) and compute a sample statistic (for example, the sample mean $\bar{x}$) as an **estimator** of μ.
- The sample mean is a **point estimator** with some nice properties (unbiased, efficient, consistent).
- A **confidence interval** is a **interval estimator**: it is an interval which has a fixed probability of containing the true population parameter.
- Example: instead of reporting just $\bar{x} = 50$, we say that there is a 95% probability that the interval $[47, 53]$ will contain the true μ.

HOW DO WE CONSTRUCT A CONFIDENCE INTERVAL? (I/II)

- We want to contruct an interval, using our **point estimator** $\bar{x}$, with a certain probability of containing the true population mean μ.

- Starting point: **Central Limit Theorem**

$$\bar{X} \sim \mathcal{N}\left(\mu, \frac{\sigma^2}{n}\right) \quad \text{then} \quad Z := \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim \mathcal{N}(0, 1)$$

- We set the **confidence level** $(1 - \alpha)$ of our interval, which is the probability that it will contain the unobservable population mean (e.g., $1 - \alpha = 0.95$)

- We know from the standard normal distribution that:

$$P(-1.96 \leq Z \leq 1.96) = 0.95$$

HOW DO WE CONSTRUCT A CONFIDENCE INTERVAL? (II/II)

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STANDARD NORMAL TABLE

$(1-\alpha)\%$	α	$\alpha/2$	Table Look For	$z_{\alpha/2}$
90%	.10	.05	0.95	1.645
95%	.05	.025	0.975	1.96
98%	.02	.01	0.99	2.33
99%	.01	.005	0.995	2.58

QUESTION 1

Q1: For a given confidence level with known σ, consider the **margin of error** in a confidence interval for the population mean. [...]

Theory Needed for Q1:

- The **margin of error (E)** is the "radius" of a confidence interval, meaning it's the amount added and subtracted from a sample statistic to create the interval: $[\bar{X} - E, \bar{X} + E]$.
- Given the definition of **confidence interval** derived before, the margin of error is given by:

$$E = z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

- The **margin of error depends** on:
  - The chosen **confidence level** (through $z_{\alpha/2}$)
  - The **population standard deviation** σ (assummed to be known).
  - The **sample size** n (larger n reduces E)

QUESTION 1

Q1: For a given confidence level with known σ, consider the **margin of error** in a confidence interval for the population mean.

Task: State which of the following statements is correct:

(A) The margin of error is the same for all samples of the same size.

(B) The margin of error is independent of sample size.

(C) The margin of error increases as the sample size increases.

(D) The margin of error varies from sample to sample of the same size.

QUESTION 2

Q2: A 95% **confidence interval** for a population mean was reported to be $[148.69,\ 155.31]$. If $\sigma = 15$, **determine the sample size** n used. Round your answer to the nearest integer.

Task: Identify the margin of error, the critical value, and **solve for** n.

QUESTION 3

Q3: Which changes make a confidence interval for the mean **narrower**?

Task: Check all that apply and explain why.

- Increasing the sample size n.
- Increasing the confidence level.
- Increasing the sample mean (holding other quantities fixed).
- A smaller population standard deviation σ.

QUESTION 4

Q4: A preliminary study reported the **standard deviation** of preview time at movie theaters is $\sigma = 6$ minutes. Assume 95% confidence.

1. If we want to estimate the population mean for preview time at movie theaters with a **margin of error of 105 seconds**, what **sample size** should be used?
2. If we want to estimate the population mean for preview time at movie theaters with a **margin of error of 1 minute**, what **sample size** should be used?

QUESTION 5

Q5: A simple random sample of $n = 50$ items from a population with $\sigma = 8$ yields a sample mean $\bar{x} = 32$.

1. Compute the 90% **confidence interval** for the population mean μ.
2. Compute the 99% **confidence interval** for the population mean μ.
3. Describe how the margin of error and interval length change as $(1 - \alpha)$ increases.

WHEN σ IS UNKNOWN?

- In practice we usually **don't know the population variance σ^2**. We estimate it with the sample variance estimator S^2.

- **New standardization**: we will use the new statistic T that is approximately distributed as **student t-distribution** with $n-1$ degrees of freedom:

$$T = \frac{\bar{X} - \mu}{s/\sqrt{n}} \overset{appr.}{\sim} t_{n-1}$$

- The student-t distribution is centered at 0 like $N(0,1)$ but has **heavier tails** to reflect extra uncertainty from estimating σ. As the sample size grows, it **converges to the normal** distribution.

- The new **confidence interval** is computed as

$$\bar{x} \pm t_{\alpha/2,\, n-1} \frac{s}{\sqrt{n}}$$

QUESTION 6

Q6: Which statements about the **Student t-distribution** are **true**? Provide a short justification (assume σ is unknown)

1. If X is **normally distributed** and the **sample size is small**, we **should not use** t-critical values to compute a CI for the mean.
2. As sample size increases, the Student-t distribution becomes **more similar** to the **standard normal**.
3. $\frac{\bar{X}-\mu}{\sigma/\sqrt{n}}$ is distributed **exactly** **as Student-t** even if X **is not normal**.
4. If the **population standard deviation** σ is **known**, we should use t-critical values to compute a confidence interval for the mean.

QUESTION 7

Q7: A simple random sample of $n = 90$ items gives $\bar{x} = 60$ with population standard deviation $\sigma = 15$.

1. Compute the 95% **confidence interval** for μ using $n = 90$.
2. Suppose the same sample mean were obtained with **sample size** $n = 180$. Compute the 95% **confidence interval**.

QUESTION 8

Q8: A feed supplement is tested on $n = 121$ cows: the **sample mean weight gain** is $\bar{x} = 47$ pounds. A 95% **confidence interval** for the mean weight gain has **margin of error** $E = 8$ pounds. Which of the following statements is a valid interpretation of the 95% confidence interval?

- It is estimated that the **mean weight gain** of cows fed this supplement **is between** 39 and 55 pounds, and the **estimation procedure succeeds** 95% of the time.
- 95% of the 121 cows studied **gained between** 39 **and** 55 **pounds**.
- If **another sample of 121 cows is tested**, there is a 95% chance their average will fall between 39 and 55 pounds.
- We are 95% sure that the **sample average** among these 121 cows **is between** 39 and 55 pounds.