ECON220B Discussion Section 7

Local Averate Treatment Effect

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INTRODUCTION TO LATE

Potential Outcome

Consider a simple setting where y_i is hourly earning and x_i is the partecipation decision in a job traning program:

$$y_i = x_i y_i(1) + (1 - x_i) y_i(0)$$

$$x_i = \mathbb{1}\{\text{participation job training program}\} \in \{0, 1\}$$

We can rewrite observed outcome outcome in terms of the **average treatment effect**:

$$y_i(1) =$$

$$y_i(0) =$$

$$y_i \quad =$$

ENDOGENEITY ISSUE

- **Unobserved Potential Benefit**

  Average treatment effect is identified if $x_i \perp y_i(1), y_i(0)$. Difficult to believe the assumption in this context: the participation decision might depend on the **unobserved benefits**.

  Remember: $x_i \perp y_i(1), y_i(0)$ is equivalent to $x_i \perp u_i(1), u_i(0)$

DOES 2SLS IDENTIFY ATE?

- **Instrumental Variable**

  We introduce an **instrument**, z_i, the distance from the training center, we also specify a functional form for the treatment decision:

$$x_i = \mathbb{1}\{\tau_{ATE} + u_i(1) - u_i(0) - z_i \geq 0\}$$
$$y_i = \beta x_i + u_i$$

- **IV Conditions**

  Assume the instrument is **independent** from the unobserved potential benefit of the program, $z_i \perp u_i(1), u_i(0)$. Are the **IV conditions** satisfied?

EXOGENEITY FAILS

Although we impose the **independence** of the instrument from the unobserved potential benefit of the program, the standard IV **exogeneity condition fails**

Then, what is the **TSLS identifying**?

$E[z_i u_i] =$

STANDARD THRESHOLD CROSSING MODEL

- **Bynary Instrument**

  We are going to consider a **binary instrument** z_i that indicates whether unit i has randomly received monetary compensation to participate in the program.

- **Potential Treatment**

  Therefore, the **observed participation status** can be written as $x_i = z_i x_i(1) + (1 - z_i) x_i(0)$.

  In this example, $x_i(1)$ will represent the ith individual's participation decision if she has been randomized to receive the compensation.

  We also specify the treatment take-up decision rule: $x_i = \mathbb{1}\{\alpha_{0i} + \alpha_1 z_i \geq v_i\}$

DIFFERENT TREATMENT TAKE-UP DECISIONS

Note that we can divide individuals into **four categories**:

1. **Always-takers**: $x_i(1) = x_i(0) = 1$
2. **Compliers**: $x_i(1) = 1, x_i(0) = 0$
3. **Defiers**: $x_i(1) = 0, x_i(0) = 1$
4. **Never-takers**: $x_i(1) = x_i(0) = 0$

Can we rule out certain categories by imposing additional assumptions on the functional form of the treatment take-up decision?

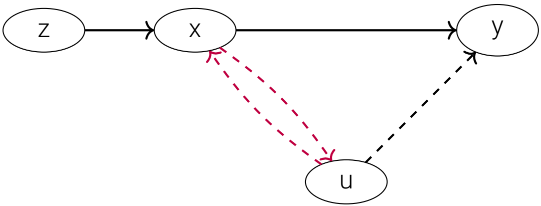
ESSENTIAL TOOLS

Must-have for LATE framework and final exam:

1. x_i binary, therefore $x_i(1)$ and $x_i(0)$ binary.
2. $E[x_i(0)] =$
3. $z_i x_i =$
4. Chain rule: $E[z_i x_i] =$
5. Law of total probability: $P(x_i = 1) =$

IDENTIFICATION ASSUMPTIONS (1/4)

We have a **binary instrument**, z_i, meaning that we have two potential treatments and four potential outcomes. How can we formulate the **exclusion restriction**?



IDENTIFICATION ASSUMPTIONS (2/4)

We can also reformulate the **exogeneity assumption**:

$$z_i \perp (y_i(1), y_i(0), x_i(1), x_i(0))$$

1. This assumption implies $z_i \perp (u_i(1), u_i(0))$. Whyt?
2. z_i will always be correlated with x_i. Why?

IDENTIFICATION ASSUMPTIONS (3/4)

What does it mean that z_i is **relevant** in this **binary setting**? The instrument is relevant if receiving the monetary compensation changes the **probability** of going into **treatment**.

$$P(x_i = 1 | z_i = 1) \neq P(x_i = 1 | z_i = 0)$$

$\text{Cov}(z_i, x_i) =$

IDENTIFICATION ASSUMPTIONS (3/4)

IDENTIFICATION ASSUMPTIONS (4/4)

Monotonicity: if the instrument can change some individuals' tratment decisions, they will be affected in the same direction:

$$\text{either} \quad x_i(1) \geq x_i(0) \quad \text{or} \quad x_i(1) \leq x_i(0) \quad \forall i$$

LATE THEOREM

Consider assumptions (1)-(4), then the **TSLS slope estimator** is consistent for:

$$\hat{\beta}_{IV} \xrightarrow{p} \frac{\text{Cov}(y_i, z_i)}{\text{Cov}(x_i, z_i)} = E[y_i(1) - y_i(0)|x_i(1) > x_i(0)]$$

The probability limit is the **Local Average Treatment Effect**, the causal effect of treatment on a specific subpopulation, **the compliers**.

DENOMINATOR $\hat{\beta}_{IV}$

$\text{Cov}(z_i, x_i) = [\, P(x_i(1) = 1) - P(x_i(0) = 1)\,]\, E[z_i]E[1 - z_i]$

NUMERATOR $\hat{\beta}_{IV}$

$\text{Cov}(z_i, y_i) =$

PUT IT ALL TOGETHER

$\hat{\beta}_{IV} =$

LET'S MAKE LATE OPERATIONAL

1. We just proved that the **probability limit** of the TSLS is the average causal effect on the compliers:

$$\hat{\beta}_{IV} \xrightarrow{p} E[y_i(1) - y_i(0)|x_i(1) > x_i(0)]$$

2. We want to express the probability limit in terms of **observables**:

$$E[y_i(1) - y_i(0)|x_i(1) > x_i(0)] = \frac{E[y_i|z_i = 1] - E[y_i|z_i = 0]}{E[x_i|z_i = 1] - E[x_i|z_i = 0]}$$

LET’S MAKE LATE OPERATIONAL