ECON220B Discussion Section 7 and 8
Instrumental Variables

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Roadmap

1. Introduction
2. Tools for IV in the Potential Outcomes Framework
3. IV and Causality
4. Local Average Treatment Effect

Lecture 7 - Introduction to IV

Motivation (1/2)

We studied the following linear regression model: $y_i = \beta_0 + \beta_1 x_i + u_i$

1. $E[u_i|x_i] = 0$

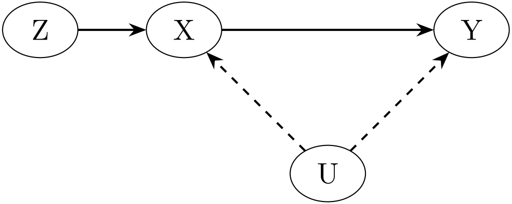
2. $E[u_i x_i] = 0$

3. $E[u_i x_i] \neq 0$

Motivation (2/2)

We can solve the endogeneity issue if we have a valid instrument z_i:

1. $E[z_i u_i] = 0$
2. $E[z_i x_i] = \alpha \neq 0$



Four Differerent Estimators

Method of Moments estimator:

$E[z_i u_i] = 0$

$E[u_i] = 0$

Four Differerent Estimators

Wald estimator: we can estimate the parameter of interest using two reduced form auxiliary regression.

$\hat{\beta}_1 = \frac{\overline{zy} - \bar{z}\bar{y}}{\overline{zx} - \bar{z}\bar{x}} =$

Four Differerent Estimators

2SLS estimator: we want to extract part of x_i that is uncorrelated with u_i.

Four Differerent Estimators

Control Function approach: we try to separate the error u_i into parts that are correlated and uncorrelated with x_i.

$$u_i = \theta v_i + \varepsilon_i$$

What Are We Doing?

- Suppose we have a valid instrument z_i:
  1. $E[z_i u_i] = 0$
  2. $E[z_i x_i] = \alpha \neq 0$

- We are able to consistently estimate the parameter of interest $\hat{\beta} \xrightarrow{p} \beta_1$.

- How can we interpret it?

Lecture 8 - LATE

Introduction to LATE

Potential Outcome framework:

- $y_i = x_i y_i(1) + (1 - x_i) y_i(0)$
- $x_i = \mathbb{1}\{\text{participation job training program}\} \in \{0, 1\}$

We can rewrite individual outcome in terms of τ_{ATE}:

- $y_i(1) =$
- $y_i(0) =$
- $y_i \quad =$

Why Endogeneity Issue

- Average Treatment Effect identified if $x_i \perp y_i(1), y_i(0)$
- Difficult to believe the assumption in this context: participation decision might depend on unobserved benefit of participating in the program:

$$y_i(1) - y_i(0) = \tau_{ATE} + u_i(1) - u_i(0)$$

Instrumental Variable Approach

- Let's introduce the following instrument, z_i, as the distance from the training center. Moreover, we will specify a functional form for the treatment variable:

$$x_i = \mathbb{1}\{\tau_{ATE} + u_i(1) - u_i(0) - z_i \geq 0\}$$

- Assume the instrument is independent from the benefit of the program, i.e. $z_i \perp u_i(1), u_i(0)$. Does the exogeneity condition hold?

- Necessary conditions: relevance $Cov(z_i, x_i) = \alpha \neq 0$ and exogeneity $E[z_i u_i] = 0$.

Exogeneity Fails

Although we impose independence of the instrument from the benefit of the program, we have no guarantee that the exogeneity condition will hold.

$E[z_i u_i] =$

Lecture 8 - Threshold Crossing Model

LATE in Threshold Crossing Model

- We are going to consider a binary instrument z_i that indicates whether unit i has received monetary compensation to participate in the program. In other words, individuals are randomized to receive a small monetary compensation

- Therefore, the observed partecipation status can be written as $x_i = z_i x_i(1) + (1 - z_i) x_i(0)$

- Now we specify the treatment take-up decision rule: we can rewrite $x_i = \mathbb{1}\{\alpha_{0i} + \alpha_1 z_i \geq v_i\}$

Treatment Take-Up Decisions & Subpopulation

Note that we can separate our observations into four groups:

1. Always-takers: $x_i(1) = x_i(0) = 1 \longrightarrow \alpha_{0i} > v_i$
2. Compliers: $x_i(1) = 1, x_i(0) = 0 \longrightarrow \alpha_{0i} < v_i < \alpha_{0i} + \alpha_1$
3. Defiers: $x_i(1) = 0, x_i(0) = 1 \longrightarrow$ **IMPOSSIBLE**
4. Never-takers: $x_i(1) = x_i(0) = 0 \longrightarrow \alpha_{0i} + \alpha_1 < v_i$

Given our assumptions on z_i and α_1 we can rule out defiers.

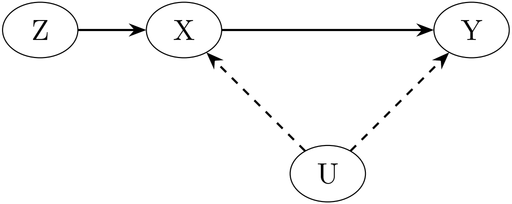
Essential Tools

Must-have for LATE framework and final exam:

1. x_i binary, therefore $x_i(1)$ and $x_i(0)$ binary.
2. $E[x_i(0)] =$
3. $z_i x_i =$
4. Chain rule: $E[z_i x_i] =$
5. Law of total probability: $P(x_i = 1) =$

Identification Assumptions (1/4)

We have a binary instrument, z_i, meaning that we have two treatments and four potential outcomes. First assumption that we need is exclusion restriction:



Identification Assumptions (2/4)

Then we impose a different exogeneity assumption:

$$z_i \perp (y_i(1), y_i(0), x_i(1), x_i(0))$$

1. This assumption implies $z_i \perp (u_i(1), u_i(0))$. Why?
2. z_i will always be correlated with x_i. Why?

Identification Assumptions (3/4)

We have a new relevance condition:

$$P(x_i = 1|z_i = 1) \neq P(x_i = 1|z_i = 0)$$

In other words, instrument has an impact in terms of treatment take-up decision.

$Cov(z_i, x_i) =$

Identification Assumptions (3/4)

Identification Assumptions (4/4)

Monotonicity: if the instrument is able to change some individuals' decisions, they will be affected in the same direction:

$$\text{either } x_i(1) \geq x_i(0) \text{ or } x_i(1) \leq x_i(0) \;\; \forall i$$

LATE Theorem

Under assumptions (1)-(4) we have

$$\hat{\beta}_{IV} = \frac{Cov(y_i, z_i)}{Cov(x_i, z_i)} \xrightarrow{p} E[y_i(1) - y_i(0) | x_i(1) > x_i(0)]$$

that is why it is called Local Average Treatment Effect: it identifies causal effect for a specific subpopulation.

Denominator $\hat{\beta}_{IV}$

$Cov(z_i, x_i) = [P(x_i(1) = 1) - P(x_i(0) = 1)] \, E[z_i]E[1 - z_i]$

Numerator $\hat{\beta}_{IV}$

$Cov(z_i, y_i) =$

Put It All Together

$\hat{\beta}_{IV} =$

Let's Make LATE Operational

1. We just proved that:

$$\hat{\beta}_{IV} = \frac{Cov(y_i, z_i)}{Cov(x_i, z_i)} \xrightarrow{p} E[y_i(1) - y_i(0)|x_i(1) > x_i(0)]$$

2. We cannot put an hat. We will now prove that:

$$E[y_i(1) - y_i(0)|x_i(1) > x_i(0)] = \frac{E[y_i|z_i = 1] - E[y_i|z_i = 0]}{E[x_i|z_i = 1] - E[x_i|z_i = 0]}$$

Let's Make LATE Operational

Exercise - LATE

- In 1947, a legislative change in the UK increased the minimum school leaving age from 14 to 15, affecting children born in 1933 and after. This change in the law provides an opportunity to evaluate the effect of (additional) schooling on earnings.

- Estimates indicate that the reform increased the average years of schooling for men by 0.397 years.

- Other estimates indicate that one extra year of schooling leads to 2% higher wages, while the reform was associated with an overall increase in wages of 1.3%.

Exercise - LATE

Our linear model is:

$$log(\text{WAGE})_i = \beta_0 + \beta_1 \text{SCHOOLING}_i + u_i$$

(1) Describe a possible source of endogeneity.

(2) Where is $\hat{\beta}_{OLS}$ converging to? What is the sign of the bias?

(3) Can we satisfy the four assumptions required to identify LATE?

(4) Obtain $\hat{\beta}_{IV}$ and compare your result with $\hat{\beta}_{OLS}$.