ECON220C Discussion Section 3 Time and Individual Fixed Effect

Lapo Bini

- Consider a panel data model with **person specific** and **time fixed** effects , where we observe each individual for two periods: $y_{it} = x_{it}\beta + \gamma_t + \alpha_i + u_{it}$.
- Here α_i is the unobserved person specific shock, γ_t is the time specific shock, and u_{it} an unobserved time and person specific shock and $x_{it} = (x_{1it} \dots x_{kit})'$.
- Assume $\{Y_i, X_i, \alpha_i, U_i\}_{i=1}^n$ iid, U_i independent of $(X_i, \gamma_t, \alpha_i)$ and $E[u_{it}] = E[\alpha_i] = 0$, all the random variables have finite second moments.

Questions

- (i) Are first difference and within approach equivalent?
- (ii) Is there a way to replace time fixed effects with dummies?
- (iii) Identify β using first difference approach. Show that β is identified under the condition $E[\Delta x_{it}(\Delta x_{it} E[\Delta x_{it}])']$ full rank.
- (iv) Consider running an OLS regression with a constant on the first difference, i.e. $\tilde{x}_{it} = (1\Delta x_{1it} \dots \Delta x_{kit})'$. Let:

$$(\hat{\kappa}\,\hat{\beta}) = \underset{(k,b)}{\operatorname{arg\,min}} \,\,\frac{1}{n}\sum_{i=1}^{n}\left(\Delta y_{it} - k - \Delta x'_{it}b\right)^{2}$$

characterize joint asymptotic distribution of the two estimators under full rank assumption.

(v) Propose two estimators for γ_1 and γ_2 .



Answer

Model

 $y_{it} = x_{it}\beta + z_i\gamma + \alpha_i + u_{it}$ where $U_i|X_iZ_i \sim \mathcal{N}(0, \sigma_u^2 I_T), U_i$ is independent of α_i and x_{it} is a scalar.

Questions

- (i) If $\alpha_i | X_i z_i \sim \mathcal{N}(0, \sigma_{\alpha}^2)$, are β and γ identified?
- (ii) If α_i is correlated with x_{it} and z_i , are β and γ identified?
- (iii) Suppose $x_{it} = (x_{1it}x_{2it}), z_i = (z_{1i}z_{2i}), (2 \text{ time-invariant factors})$ where x_{1it} is correlated with z_{1i} , and x_{2it}, z_{2i} are uncorrelated with. Are β and γ identified?

Qual Question 2016

Answer