Practice Test

Static & Dynamic Panel

Question 1

Consider the following static panel data model:

$$y_{it} = x_{it}\beta + \varepsilon_{it}$$

for $t = 1, \dots, T$ and $i = 1, \dots, N$. Assume that $\{X_i, Y_i, \}_{i=1}^{n}$ is *iid*.

(a) Under what condition(s) is $\hat{\beta}_{POLS}$ consistent for β for a fixed T as $N \to \infty$? How would you estimate the asymptotic variance of $\hat{\beta}_{POLS}$?

(b) Assume now $\epsilon_{it} = \alpha_i + u_{it}$ what assumption do you need to have a standard random effect model? Is $\hat{\beta}_{POLS}$ consistent for β for a fixed T as $N \to \infty$? Is it efficient?

(c) Under what condition(s) on α_i we have a fixed effect model? Propose a consistent estimator for β under FE model, state clearly under what conditions it is consistent and prove it.

(d) Consider now $y_{git} = x_{git}\beta + \varepsilon_{git}$, where $\varepsilon_{git} = \alpha_i + \lambda_t + \mu_g + u_{git}$. Group, individual and time fixed effects are mutually uncorrelated, and independent from the error term $u_{git} \sim (0, \sigma_u^2)$ $\forall g, i, t$. Stack all the observations across time $t = 1, \dots, T$ for each individual i into $Y_{gi} = X_{gi}\beta + \epsilon_{gi}$, derive an analytical expression for the variance covariance matrix $Var(\epsilon_{gi}) = \Omega_i$. Impose extra conditions if you need them. Derive an analytical expression for $Cov(\epsilon_{gi}, \epsilon_{g'i'})$.

(e) Propose a strategy to get rid of the three fixed effects and get $\tilde{y}_{git} = \tilde{x}_{git}\beta + \tilde{u}_{git}$.

(f) Propose an estimator for the model $\tilde{y}_{git} = \tilde{x}_{git}\beta + \tilde{u}_{git}$. Under what condition(s) is your estimator consistent for β for a fixed T and for a fixed number of individuals within group as $G \to \infty$? Explain how you would estimate the asymptotic variance of your estimator and provide an analytical expression.

Question 2

Consider the dynamic panel data model:

$$y_{it} = y_{it-1}\beta + \alpha_i + u_{it}$$
$$u_{it} = e_{it} + \theta e_{it-1}$$

for $i = 1, \dots, n$ and $t = 1, \dots, T$ where conditional on $\{y_{i0} \ : \ i = 1, \dots, n\}$ we have $e_{it} \sim iid(0, \sigma_e^2)$ across i and t, α_i is independent across i with variance $\sigma_{\alpha_i}^2 > 0$ and uncorrelated with $e_{i1}, \dots, e_{iT}$. Assume that $\sigma \neq 0$ and consider the asymptotics as $n \to \infty$ for a fixed T.

(a) State what strict exogeneity is and why it fails in those types of dynamic panel data setting. Then state what sequential exogeneity is and interpret it.

(b) Can we consistently estimate β using FD estimator?

(c) In the absence of the weak instrument problem, is y_{it-2} a valid instrument for the FD equation: $y_{it} - y_{it-1} = (y_{it-1} - y_{it-2})\,\beta + (u_{it} - u_{it-1})$?

(d) Suppose $T = 3$, is the simple ARMA(1,1) model just identified, exactly identified or over identified? (Hint: check if there are more instruments than the endogenous regressors).

(e) Suppose $T = 4$, design an AB style GMM estimator of β that is as asymptotically efficient as possible, assume that any exogenous instruments you find are also strongly relevant so that there will be no weak instrumentation problems). Please be specific about the form and the number of moment conditions. Please write down the GMM objective function.

(f) When does the AB estimator fail?