ECON220C Discussion Section 7
Probit Model

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Roadmap

1. Probit & Linear Projection
2. Probit & MLE
3. Probit & IV
4. Probit & ULLN

Probit Model

Consider the following **probit model**:

$$x_i = \mathbb{1}\{z_i\gamma + v_i > 0\}$$

assume that z_i and v_i are **independent standard normal** and where $\mathbb{1}\{\cdot\}$ is the indicator function.

Questions

- We aim to estimate the linear projection of x_i onto z_i. Write down the moment conditions to estimate an intercept a and a slope coefficient b. What is the problem with this approach?
- Derive the probability limit of the intercept $a^\star$ and slope estimator $b^\star$. *Hint: Intercept is equal to a number, slope depends only on z.*
- Show that $(b^\star)^2 \leq 1/3$ applying Cauchy-Schwarz inequality and property CDF.

Probit & MLE

Consider the following **probit model**:

$$x_i = \mathbb{1}\{z_i\gamma + v_i > 0\}$$

assume that z_i and v_i are **independent standard normal**. We can write the **linear projection** studied before as $x_i = L[x_i|z_i] + e_i$ where $L[x_i|z_i] = a^\star + b^\star z_i$.

Questions

- Show that $Var(x_i) = Var(L(x_i|z_i)) + E[(x_i - L[x_i|z_i])^2]$
- Write down the log-likelihood function to estimate γ and derive the first order condition.
- Show that $Var(E[x_i|z_i]) \geq Var(L[x_i|z_i])$

Probit & IV

Consider the following **structural model**:

$$y_i = \alpha + x_i\beta + u_i$$

where x_i is a dummy variable. Assume that $x_i = \mathbb{1}\{z_i\gamma + v_i > 0\}$ where z_i is independent of u_i and v_i, $E[u_i|z_i] = E[v_i|z_i] = 0$, $E[u_i|z_i, v_i] = \rho v_i$ for some $\rho \neq 0$.

Questions

- Describe the endogeneity problem here. Define the Wald IV estimator $\hat{\beta}_{IV}$. Do you need any distributional assumption on v_i to prove consistency? Prove consistency.
- Derive $E[x_i|z_i]$ under the assumption of v_i standard normal. Write down the TSLS estimator taking into account the conditional expectation function.
- Is your TSLS estimator consistent? Prove it.

Probit & ULLN

Consider the following **structural model**:

$$y_i = \alpha + x_i\beta + u_i$$

where x_i is a dummy variable. Assume that $x_i = \mathbb{1}\{z_i\gamma + v_i > 0\}$ where z_i is independent of u_i and v_i, $E[u_i|z_i] = E[v_i|z_i] = 0$, $E[u_i|z_i, v_i] = \rho v_i$ for some $\rho \neq 0$.

Questions

- Given $\sqrt{n}\left(\hat{\beta}_{IV} - \beta\right) = \frac{1}{\sqrt{n}}\sum_i Cov(z_i, x_i)^{-1} z_i u_i + o_p(1)$. Show that the asymptotic variance of $\sqrt{n}\left(\hat{\beta}_{IV} - \beta\right)$ is equal to $\sigma_u^2 Var(L[x_i|z_i])^{-1}$
- Define $w_i(d) \equiv \Phi(x_i d)$, using a uniform law of large numbers type of argument shows that $\frac{1}{\sqrt{n}}\sum_i w_i(\hat{\gamma})u_i = \frac{1}{\sqrt{n}}\sum_i w_i(\gamma)u_i + o_p(1)$
- Given $\sqrt{n}\left(\hat{\beta}_{TSLS} - \beta\right) = \frac{1}{\sqrt{n}}\sum_i Cov(w_i(\gamma), x_i)^{-1} w_i(\hat{\gamma})u_i + o_p(1)$, Find the asymptotic variance.

Answer