One Hundred Years of Business Cycles and the Phillips Curve

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How stable are business cycle regularities?

(Is there a Phillips Curve?)

Recent debate

- Stability, regime shifts & non-linearities
- Flattening and steepening of the Phillips Curve
- Inflation puzzles
- Changes in the transmission of monetary policy
- Labour market transformations
- Fed's delayed response motivated by flat-PC...

This paper:

- Long view, stylised model, fixed-parameters, reduced form approach

What are business cycle fluctuations?

Burns and Mitchell (1946)

Business cycles are a type of fluctuation found in the aggregate economic activity of nations that organize their work mainly in business enterprises: **a cycle consists of expansions occurring at about the same time in many economic activities, followed by similarly general recessions**

This sequence of changes is **recurrent but not periodic**. *In duration business cycles vary* **from more than one year to ten or twelve years**; *they are not divisible into shorter cycles of similar character with amplitudes approximating their own.*

A semi-structural approach

Intuition:

- Look for stable multivariate dynamic relationships at business cycle frequency
- ... and in particular between slack in real activity variables and nominal variables
- Long-view and subsample analysis
- Econometrically to find business cycle correlation we need to **detrend** & **denoise**

Methodology (Hasenzagl et al, 2022):

- Multivariate trend-cycle model (Harvey, 1985)
- Common cycle, long-run trends
- Multivariate restrictions informed by economic theory and...
- ... agents' expectations to extract long-run trends and cycles
- Unrestricted signs, magnitudes and frequencies
- Bayesian estimation

A semi-structural model of business cycles

A stylised model

- Technological and institutional factors determine the long-run **output potential**

$$y_t = \tau_t^y + \hat{y}_t^{gap} = \tau_t^y + \psi_t^{gap}$$

$$\tau_t^y = \mu + \tau_{t-1}^y + u_t^{\tau,y}$$

- Cyclical factors move output off its trend and determine the **output gap**

$$\psi_t^{gap} = \rho(L)\psi_t^{gap} + v_t$$

- The gap reflects into the cyclical component of unemployment via the **Okun's law**, while the unemployment rate consistent with that potential level of output, is **NAIRU**

$$u_t = \tau_t^u + \underbrace{\delta_u \hat{y}_t^{gap}}_{\hat{u}_t^{gap}}$$

A stylised model

- The gap connects to nominal variables via the **Phillips curve**

$$\pi_t = \underbrace{\tau_t^{\pi}}_{\lim_{h\to\infty} E_t \pi_{t+h}} + \underbrace{\delta_\pi \hat{y}_t^{gap}}_{\hat{\pi}_t^{gap}} + \psi_t^{epc}$$

- **Trend inflation** is the inflation rate prevailing in the absence of cyclical factors and reflects **long-term expectations**

$$\lim_{h\to\infty} E_t \pi_{t+h} = \tau_t^{\pi} = \tau_{t-1}^{\pi} + u_t^{\tau,\pi}$$

- Cost-push and supply shocks can move inflation off the PC curve
- A stylised model of idiosyncratic and common components

$$\begin{pmatrix} y_t \\ u_t \\ \pi_t \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \delta_u & 0 \\ \delta_\pi & 1 \end{pmatrix} \begin{pmatrix} \psi_t^{gap} \\ \psi_t^{epc} \end{pmatrix} + \begin{pmatrix} \tau_t^{y} \\ \tau_t^{u} \\ \tau_t^{\pi} \end{pmatrix}. \tag{1}$$

A trend-cycle model: observation equations

$$\begin{pmatrix} y_t \\ \\ \\ \\ \\ \end{pmatrix} = \begin{pmatrix} 1 \\ \\ \\ \\ \\ \end{pmatrix} \psi_t^{gap}$$

- **Output gap** informs stationary **Business Cycle** fluctuations, , ,

A trend-cycle model: observation equations

$$\begin{pmatrix} y_t \\ e_t \\ u_t \\ \\ \\ \\ \\ \end{pmatrix} = \begin{pmatrix} 1 \\ \delta_{e,1} + \delta_{e,2} L \\ \delta_{u,1} + \delta_{u,2} L \\ \\ \\ \\ \\ \\ \end{pmatrix} \psi_t^{gap}$$

- **Output gap** informs stationary **Business Cycle** fluctuations, connects to labour market variables via **Okun's law**, ,

A trend-cycle model: observation equations

$$\begin{pmatrix} y_t \\ e_t \\ u_t \\ \pi_t \\ \pi_t^c \\ F_t^{uom}\pi_{t+4} \\ F_t^{spf}\pi_{t+4} \end{pmatrix} = \begin{pmatrix} 1 \\ \delta_{e,1} + \delta_{e,2}L \\ \delta_{u,1} + \delta_{u,2}L \\ \delta_{\pi,1} + \delta_{\pi,2}L \\ \delta_{\pi^c,1} + \delta_{\pi^c,2}L \\ \delta_{uom,1} + \delta_{uom,2}L + \delta_{uom,3}L^2 \\ \delta_{spf,1} + \delta_{spf,2}L + \delta_{spf,3}L^2 \end{pmatrix} \psi_t^{gap}$$

- **Output gap** informs stationary **Business Cycle** fluctuations, connects to labour market variables via **Okun's law**, and to prices and expectations via the **Phillips curve**,

A trend-cycle model: observation equations

$$\begin{pmatrix} y_t \\ e_t \\ u_t \\ \pi_t \\ \pi_t^c \\ F_t^{uom}\pi_{t+4} \\ F_t^{spf}\pi_{t+4} \\ i_t \end{pmatrix} = \begin{pmatrix} 1 \\ \delta_{e,1} + \delta_{e,2}L \\ \delta_{u,1} + \delta_{u,2}L \\ \delta_{\pi,1} + \delta_{\pi,2}L \\ \delta_{\pi^c,1} + \delta_{\pi^c,2}L \\ \delta_{uom,1} + \delta_{uom,2}L + \delta_{uom,3}L^2 \\ \delta_{spf,1} + \delta_{spf,2}L + \delta_{spf,3}L^2 \\ \delta_{i,1} + \delta_{i,2}L + \delta_{i,3}L^2 \end{pmatrix} \psi_t^{gap}$$

- **Output gap** informs stationary **Business Cycle** fluctuations, connects to labour market variables via **Okun's law**, and to prices and expectations via the **Phillips curve**, and the nominal rates via the **policy response function**

A trend-cycle model: observation equations

$$\begin{pmatrix} y_t \\ e_t \\ u_t \\ \pi_t \\ \pi_t^c \\ F_t^{uom}\pi_{t+4} \\ F_t^{spf}\pi_{t+4} \\ i_t \end{pmatrix} = \begin{pmatrix} 1 \\ \delta_{e,1} + \delta_{e,2}L \\ \delta_{u,1} + \delta_{u,2}L \\ \delta_{\pi,1} + \delta_{\pi,2}L \\ \delta_{\pi^c,1} + \delta_{\pi^c,2}L \\ \delta_{uom,1} + \delta_{uom,2}L + \delta_{uom,3}L^2 \\ \delta_{spf,1} + \delta_{spf,2}L + \delta_{spf,3}L^2 \\ \delta_{i,1} + \delta_{i,2}L + \delta_{i,3}L^2 \end{pmatrix} \psi_t^{gap} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \tau_t^{\pi}$$

- **Output gap** informs stationary **Business Cycle** fluctuations, connects to labour market variables via **Okun's law**, and to prices and expectations via the **Phillips curve**, and the nominal rates via the **policy response function**
- Prices and expectations share a common **trend inflation**

A trend-cycle model: observation equations

$$\begin{pmatrix} y_t \\ e_t \\ u_t \\ \pi_t \\ \pi_t^c \\ F_t^{uom}\pi_{t+4} \\ F_t^{spf}\pi_{t+4} \\ i_t \end{pmatrix} = \begin{pmatrix} 1 \\ \delta_{e,1} + \delta_{e,2}L \\ \delta_{u,1} + \delta_{u,2}L \\ \delta_{\pi,1} + \delta_{\pi,2}L \\ \delta_{\pi^c,1} + \delta_{\pi^c,2}L \\ \delta_{uom,1} + \delta_{uom,2}L + \delta_{uom,3}L^2 \\ \delta_{spf,1} + \delta_{spf,2}L + \delta_{spf,3}L^2 \\ \delta_{i,1} + \delta_{i,2}L + \delta_{i,3}L^2 \end{pmatrix} \psi_t^{gap} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \tau_t^{\pi} + \begin{pmatrix} \psi_t^y \\ \psi_t^e \\ \psi_t^u \\ \psi_t^{\pi} \\ \psi_t^{\pi^c} \\ \psi_t^{uom} \\ \psi_t^{spf} \\ \psi_t^i \end{pmatrix}$$

- **Output gap** informs stationary **Business Cycle** fluctuations, connects to labour market variables via **Okun's law**, and to prices and expectations via the **Phillips curve**, and the nominal rates via the **policy response function**
- Prices and expectations share a common **trend inflation**
- Stationary idiosyncratic disturbances

A trend-cycle model: observation equations

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- **Output gap** informs stationary **Business Cycle** fluctuations, connects to labour market variables via **Okun's law**, and to prices and expectations via the **Phillips curve**, and the nominal rates via the **policy response function**
- Prices and expectations share a common **trend inflation**
- Stationary idiosyncratic disturbances
- Independent trends in output (**output potential**) and employment/unemployment (**trend employment/equilibrium unemployment**)
- ... and idiosyncratic trends (biases) in expectations

A trend-cycle model: state equations

Stationary cycles

- Stationary ARMA(2,1), can be written as

$$\begin{pmatrix} \psi_t^j \\ \psi_t^{*j} \end{pmatrix} = \rho^j \begin{pmatrix} \cos(\lambda^j) & \sin(\lambda^j) \\ -\sin(\lambda^j) & \cos(\lambda^j) \end{pmatrix} \begin{pmatrix} \psi_{t-1}^j \\ \psi_{t-1}^{*j} \end{pmatrix} + \begin{pmatrix} v_t^j \\ v_t^{*j} \end{pmatrix}, \quad \begin{pmatrix} v_t^j \\ v_t^{*j} \end{pmatrix} \sim \mathcal{N}(0, \varsigma_j^2 \, I_2)$$

Trends and 'biases'

- Unit root process with or without a drift

$$\tau_t^j = \tau_0^j + \tau_{t-1}^j + u_t^j, \quad u_t^j \sim \mathcal{N}(0, \sigma_j^2)$$

$$\mu_t^j = \mu_{t-1}^j + u_t^j, \quad u_t^j \sim \mathcal{N}(0, \sigma_j^2)$$

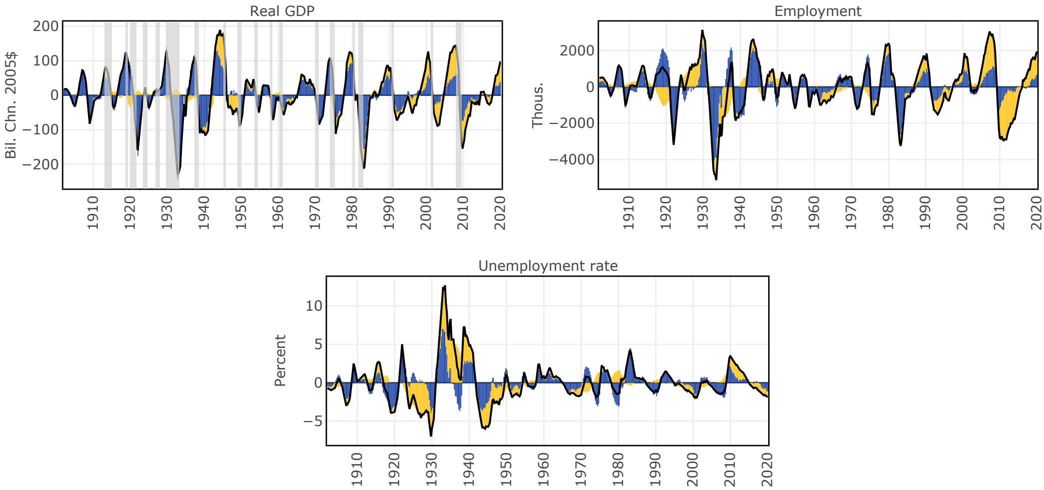
The dataset: 120 years of data

Variable	Transformation	Loads on BC Cycle	Loads on π Trend	Frequency
Gross Domestic Product	Levels	✓	✗	A1901, Q1913
Employment	Levels	✓	✗	A1901, Q1929
Unemployment Rate	Levels	✓	✗	A1901, Q1948
Inflation	YoY	✓	✓	A1914, Q1921
Core Inflation	YoY	✓	✓	Q1957
Consumers' Expected Inflation	Levels	✓	✓	Q1968
Professional Forecasters' Expected Inflation	Levels	✓	✓	S1960, Q1983
Nominal short term rate	Levels	✓	✓	A1901, Q1954

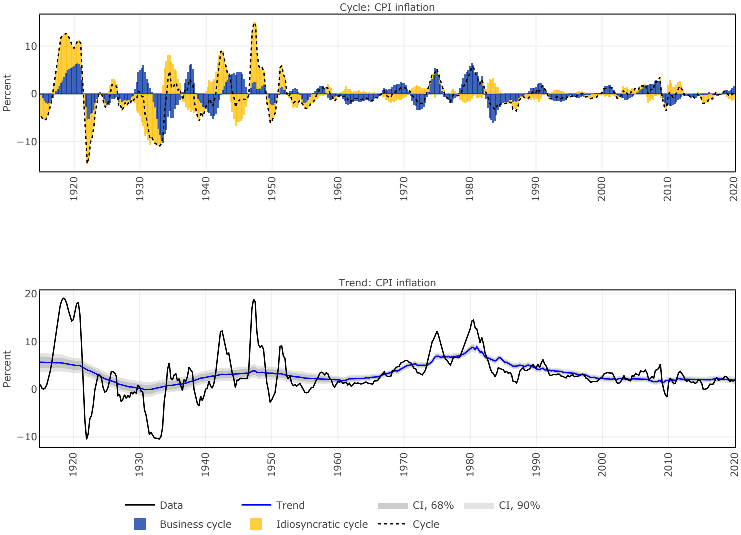
- Annually or Quarterly, Q1-1901 to Q4-2019
- Jordà-Schularick-Taylor Macrohistory Dataset
- Expectations: Livingston Survey, Philadelphia Fed SPF, University of Michigan
- Missing observations pre-WWII in survey data

Bayesian Estimation

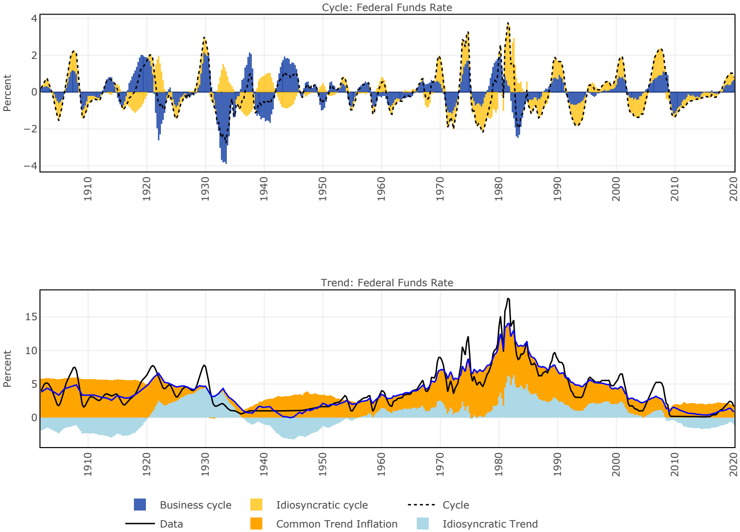
Cyclical components: real variables



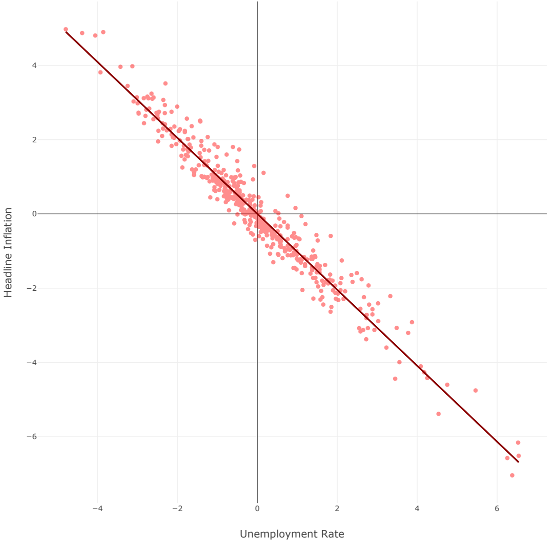
Cyclical components: inflation



Cyclical components: nominal interest rate



120 years of Phillips curve



120 years of Phillips curve

Takeaways

The long view:

- Not bad!

120 years of Phillips curve

Takeaways

The long view:

- A fixed parameters model capture much of the business cycle dynamics
- Business timing dating in line with the official NBER dates
- Much of the nominal interest rates cyclical dynamics captured by output gap
- Much of the inflation cyclical dynamics captured by output gap
- Relatively stable dynamic relationship between slack in real activity and inflation
- Relatively stable Okun's law

Post WWII business cycles (1960-2019)

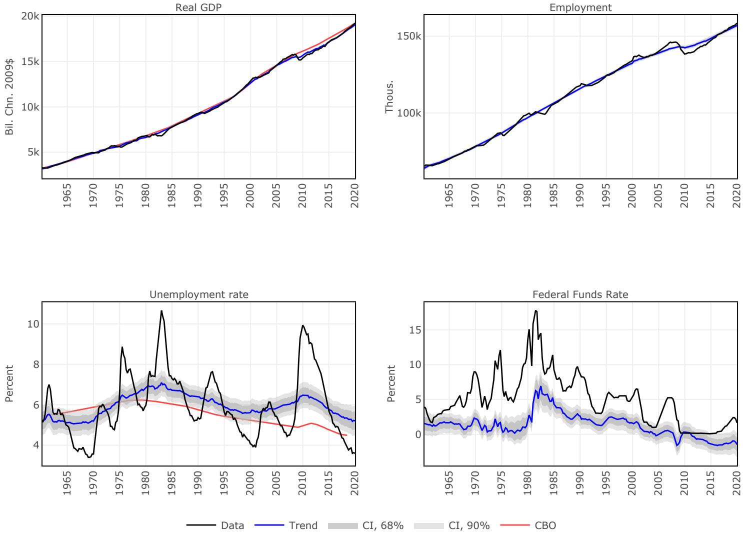
Challenges to a longer view

- National accounting is a post 1930s creation (Kuznets, 1934)
  - Quarterly data post WWII
  - … as well as expectational data (surveys)
  - … and official estimates of output gap

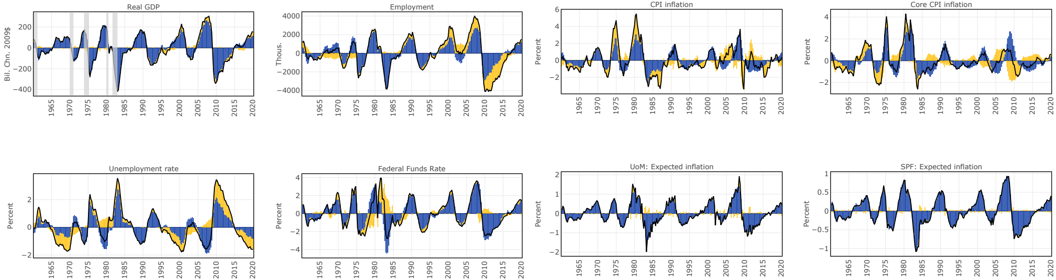
- Many regime/policy changes
  - Sterling pound gold standard (until 1913)
  - Interwar period and the Great Depression (1929-39)
  - Independence of the Fed (1951)
  - Bretton Woods system (1945-71)

- Many potential changes to the structure of the economy
  - Technology
  - Demographics
  - Labour market
  - …

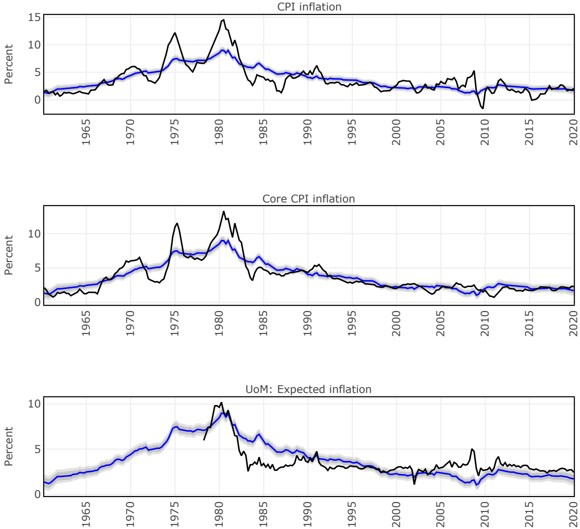
Long-run trends



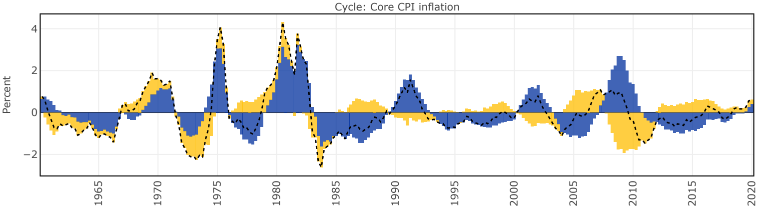
Cyclical components

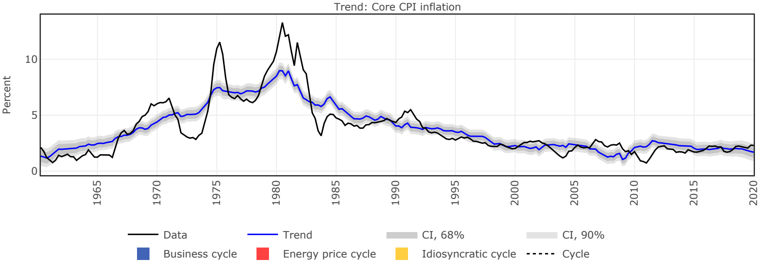


Underlying trend inflation

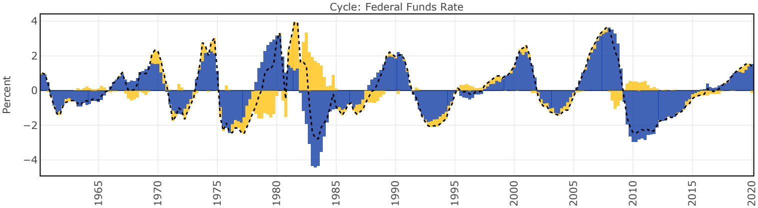


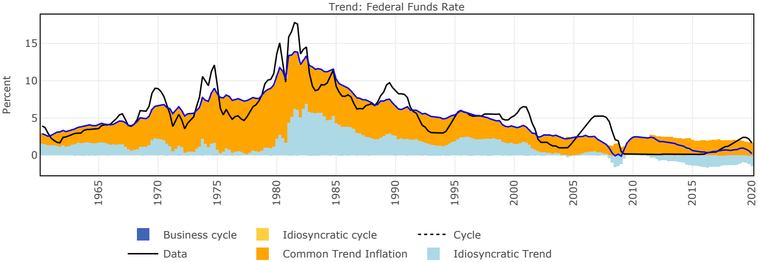
CPI inflation



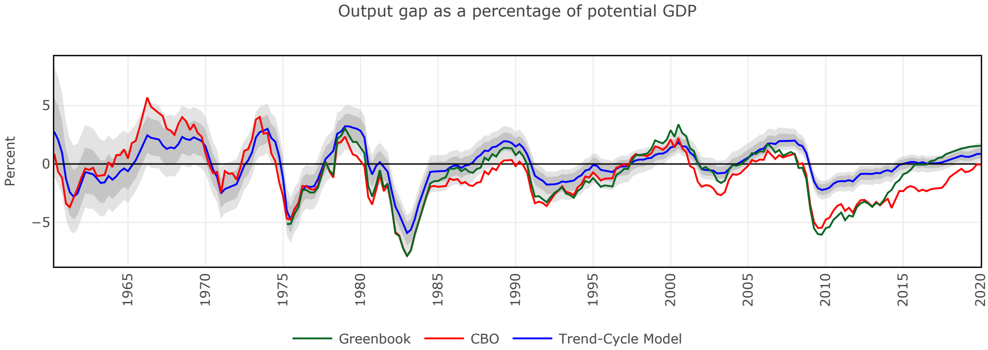


The federal funds rate

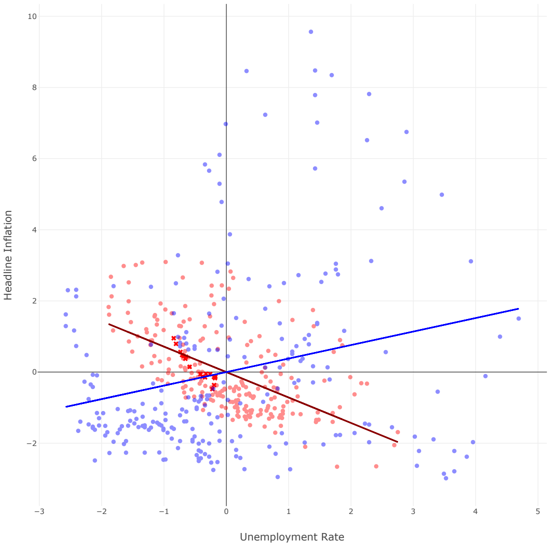




The output gap



The slope of the Phillips curve



60 years of Phillips curve

Takeaways

The (less) long view:

- Smooth trends, stable cyclical relationships
- Both inflation and interest rate have a sizeable business cycle component
- Stable positive dynamic relationship between slack in real activity and inflation
- Reduced form PC steep once data are detrended and denoised
- Potential role for supply/cost-push shocks
- Sizeable idiosyncratic component: what is it?

A model with energy price disturbances (1960-2019)

A stylised model: demand and supply

McLeay and Tenreyro (2019), Del Negro et al (2020)

$$
\begin{aligned}
(PC) \qquad & \hat{y}_t^{gap} &&= \alpha \hat{y}_{t-1}^{gap} + E_t \hat{y}_{t+1}^{gap} - \sigma(i_t - E_t \pi_{t+1} - u_t^d) \\
(IS) \qquad & \hat{\pi}_t^{gap} &&= \beta E_t \pi_{t+1} + \kappa \hat{y}_t^{gap} + u_t^s \\
(MP) \qquad & \hat{i}_t &&= E_t \pi_{t+1} + \theta_d u_t^d + \theta_\pi \hat{\pi}_t^{gap} + u_t^{mp}
\end{aligned}
$$

A stylised model: demand and supply

McLeay and Tenreyro (2019), Del Negro et al (2020)

$$
\begin{aligned}
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(IS) \qquad \hat{\pi}_t^{gap} &= \beta E_t \pi_{t+1} + \kappa \hat{y}_t^{gap} + u_t^s \\
(MP) \qquad \hat{i}_t &= E_t \pi_{t+1} + \theta_d u_t^d + u_t^{mp}
\end{aligned}
$$

If the central bank does not respond systematically to supply shocks, in equilibrium:

$$
\begin{aligned}
\hat{y}_t^{gap} &= \underbrace{\rho \hat{y}_{t-1}^{gap}}_{\rho \psi_{t-1}^{gap}} + \underbrace{\frac{\sigma((1-\theta_d)u_t^d - u_t^{mp})}{1-\rho}}_{v_t^d} \qquad (2) \\
\hat{\pi}_t^{gap} &= \underbrace{\frac{\kappa}{1-\beta\rho}}_{\delta_\pi} \hat{y}_t^{gap} + \underbrace{u_t^s}_{\psi_t^{epc}}
\end{aligned}
$$

- The central bank controls the variance of business cycle fluctuations $\theta_d \to 1$
- Flat(ter) structural Phillips curve, $\kappa = 0$

A quarterly model with energy disturbances

$$
\begin{pmatrix} y_t \\ e_t \\ u_t \\ oil_t \\ \pi_t \\ \pi_t^c \\ F_t^{uom}\pi_{t+4} \\ F_t^{spf}\pi_{t+4} \\ i_t \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \delta_{e,1}+\delta_{e,2}L & 0 \\ \delta_{u,1}+\delta_{u,2}L & 0 \\ \delta_{oil,1}+\delta_{oil,2}L & 1 \\ \delta_{\pi,1}+\delta_{\pi,2}L & \gamma_{\pi,1}+\gamma_{\pi,2}L \\ \delta_{\pi^c,1}+\delta_{\pi^c,2}L & \gamma_{\pi^c,1}+\gamma_{\pi^c,2}L \\ \delta_{uom,1}+\delta_{uom,2}L+\delta_{uom,3}L^2 & \gamma_{uom,1}+\gamma_{uom,2}L \\ \delta_{spf,1}+\delta_{spf,2}L+\delta_{spf,3}L^2 & \gamma_{spf,1}+\gamma_{spf,2}L \\ \delta_{i,1}+\delta_{i,2}L+\delta_{i,3}L^2 & \gamma_{i,1}+\gamma_{i,2}L \end{pmatrix} \begin{pmatrix} \psi_t^{gap} \\ \psi_t^{epc} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \tau_t^{\pi} + \begin{pmatrix} \psi_t^y \\ \psi_t^e \\ \psi_t^u \\ \psi_t^{oil} \\ \psi_t^{\pi} \\ \psi_t^{\pi^c} \\ \psi_t^{uom} \\ \psi_t^{spf} \\ \psi_t^i \end{pmatrix} + \begin{pmatrix} \tau_t^y \\ \tau_t^e \\ \tau_t^u \\ \tau_t^{oil} \\ 0 \\ 0 \\ \mu_t^{uom} \\ \mu_t^{spf} \\ \tau_t^i \end{pmatrix}
$$

- Oil price, prices and expectations can be affected by an ‘energy component’ (cost push shocks)

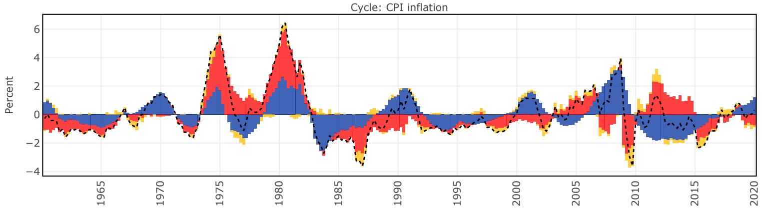
The dataset

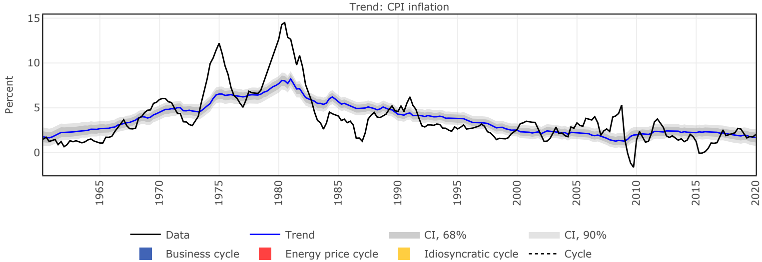
Variable	Transformation	Loads on		
		BC Cycle	EP Cycle	π Trend
Gross Domestic Product	Levels	✓	✗	✗
Employment	Levels	✓	✗	✗
Unemployment Rate	Levels	✓	✗	✗
WTI Spot Oil Price	Levels	✓	✓	✗
CPI: All Items	YoY	✓	✓	✓
Core CPI	YoY	✓	✓	✓
UoM: Expected Inflation	Levels	✓	✓	✓
SPF: Expected Inflation	Levels	✓	✓	✓
Federal Funds Rate	Levels	✓	✓	✓

- Quarterly observations Q1-1960 to Q2-2019

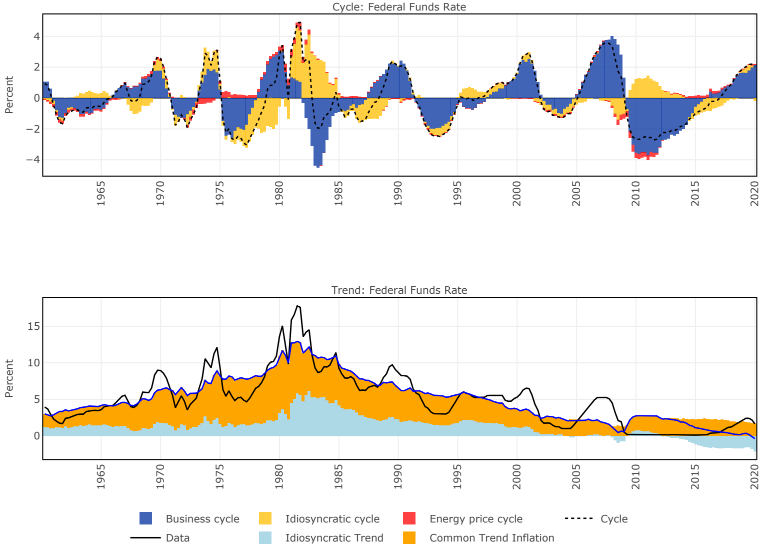
Bayesian Estimation

CPI inflation

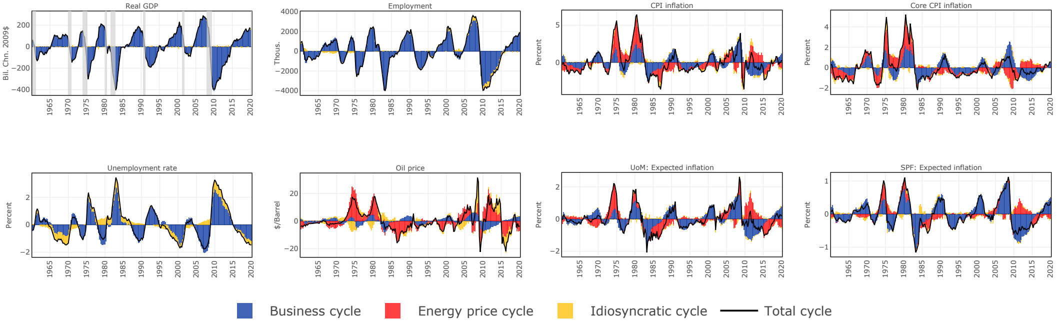




The federal funds rate



Historical decomposition



60 years of Phillips curve and oil disturbances

Takeaways

The (less) long view:

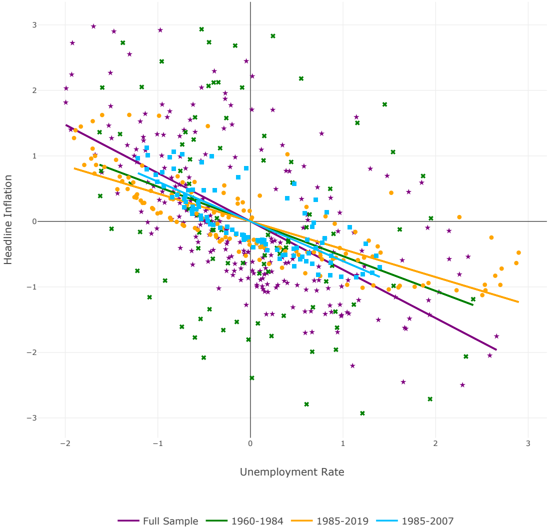
- Smooth trends, stable cyclical relationships

- Oil disturbances explain much of the noise...
- ... and potentially many of the puzzles

Did business cycles dynamics change over time?

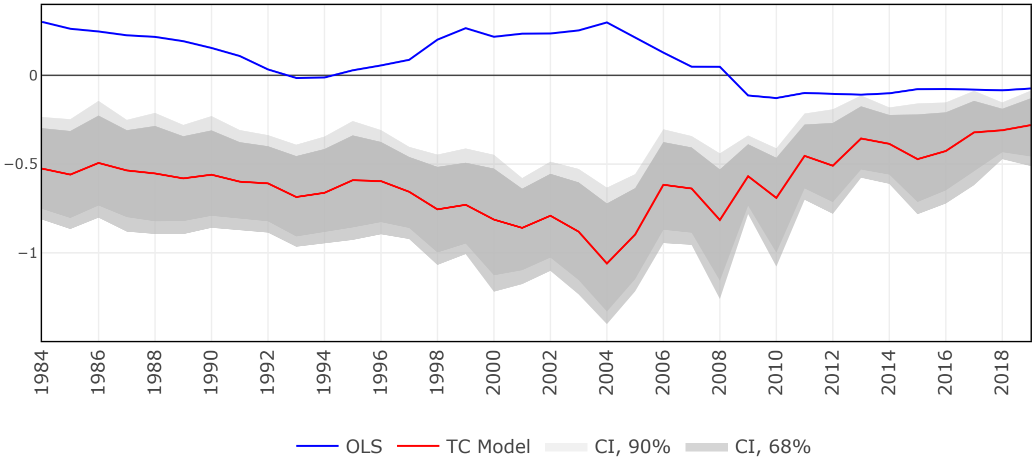
Is the Phillips curve stable?

Subsample analysis

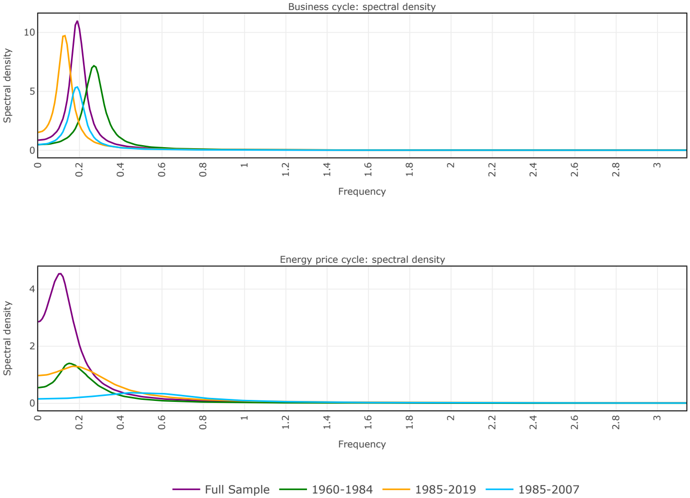


Is the Phillips curve stable?

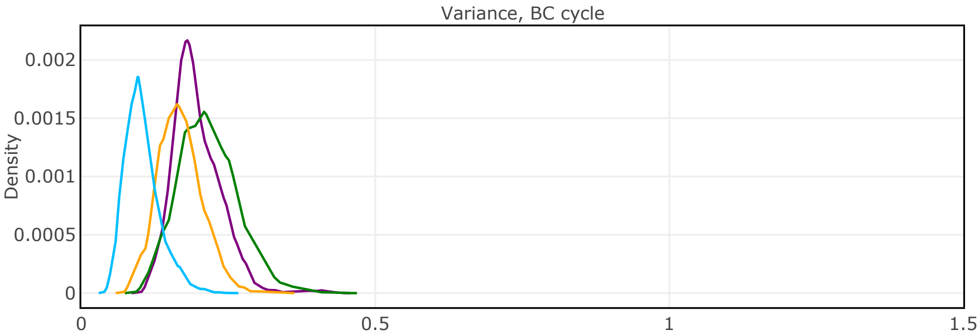
25 years rolling windows estimation



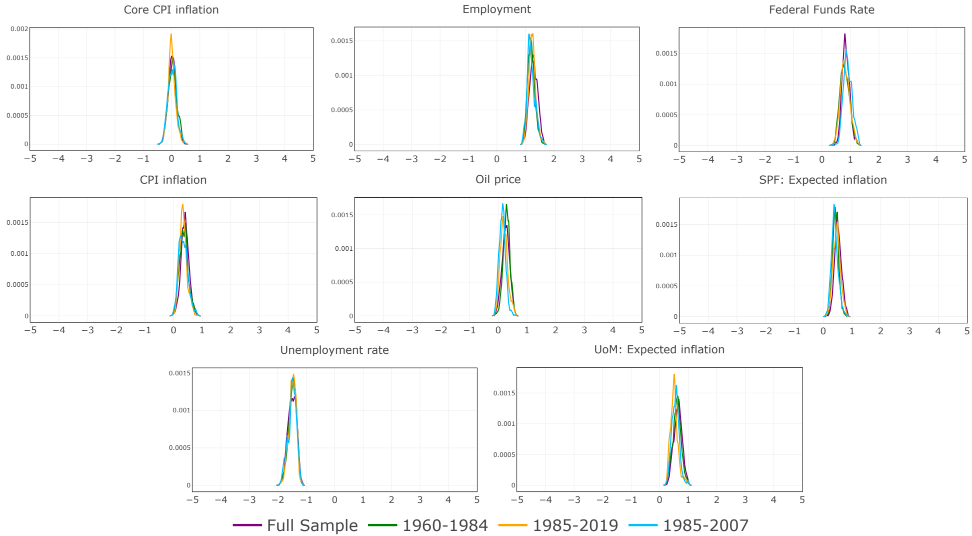
Stability: spectral densities



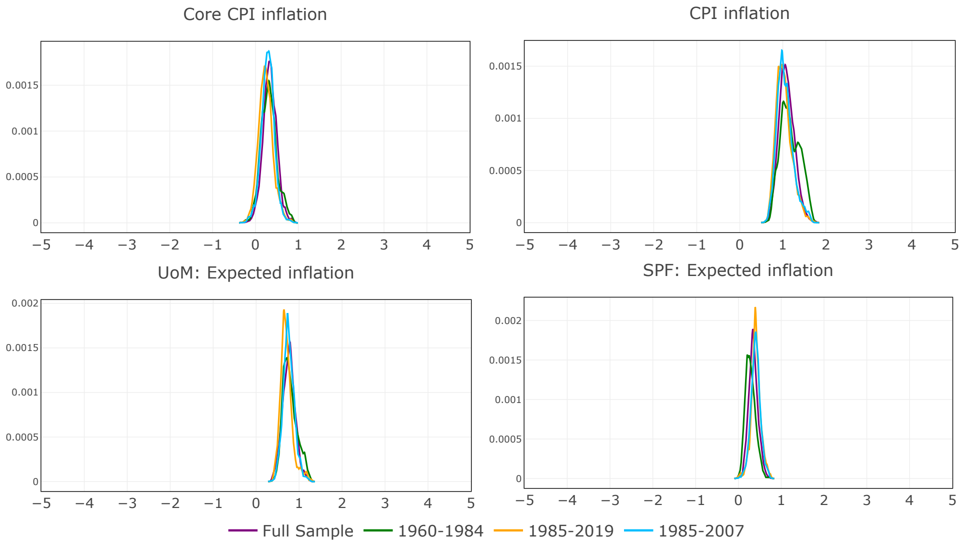
The variance of the business cycle



The business cycle loadings



Loadings energy cycle



60 years of subsample analysis

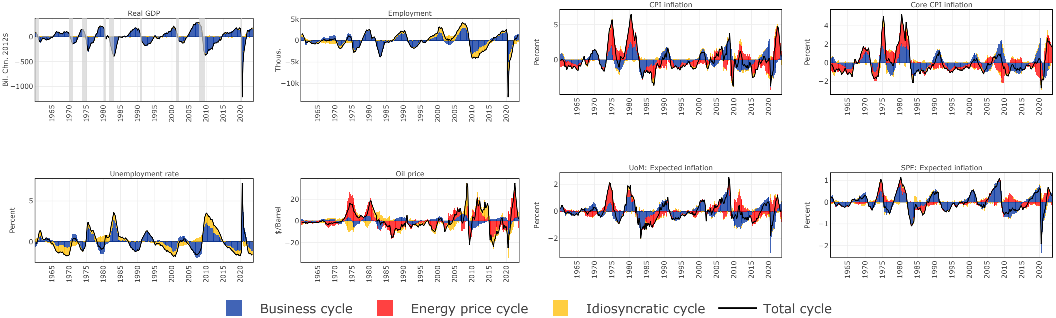
Takeaways

The (less) long view:

- Business cycle regularities stable!
- The relationship between real and nominal variables at business cycle frequency fairly stable
- Some flattening
- Monetary policy largely stable
- Variance of business cycle lower during Great Moderation: policy or luck?

COVID

Historical decomposition



COVID

Takeaways

- Fixing parameters the model fits data well
- But something may have changed...

Conclusions

Reports of the death of Phillips Curve the have been greatly exaggerated!

Appendix

Priors

Table: *Prior distributions*

Name	Support	Density	Parameter 1	Parameter 2
δ, γ, ϕ and τ	$\mathbb{R}$	Normal	0	1000
σ^2 and ς^2	$(0, \infty)$	Inverse-Gamma	3	1
ρ	$[0.001, 0.970]$	Uniform	0.001	0.970
λ	$[0.001, \pi]$	Uniform	0.001	π

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Bayesian Estimation

Metropolis-Within-Gibbs Algorithm

The algorithm is structured in two blocks

- The **first block** uses a Metropolis step for the **estimation of the state-space parameters**
- The **second block** uses a Gibbs step to draw the **unobserved states** conditional on the model parameters

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Bayesian Estimation

- Metropolis algorithm draws the model parameters in the unbounded space in order to avoid a-priori rejections and to obtain a more efficient estimation routine
- The following transformations have been applied to parameters with Normal, Inverse-Gamma and Uniform priors

$$\theta_j^N = \Theta_j^N \qquad \theta_j^{IG} = \ln(\Theta_j^{IG} - a_j) \qquad \theta_j^U = \ln\left(\frac{\Theta_j^U - a_j}{b_j - \Theta_j^U}\right)$$

Where a_j and b_j are the lower and the upper bounds for the j-th parameter

- Jacobians of the transformations of the variables

$$\ln\left(\frac{d\Theta_j^N}{d\theta_j^N}\right) = 0 \qquad \ln\left(\frac{d\Theta_j^{IG}}{d\theta_j^{IG}}\right) = \theta_j^{IG}$$

$$\ln\left(\frac{d\Theta_j^U}{d\theta_j^U}\right) = \ln(b_j - a_j) + \theta_j^U - 2\ln(1 + \exp(\theta_j^U))$$

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Bayesian Estimation

Algorithm: Metropolis-Within-Gibbs

Initialisation

For $s = 1, \dots, n_s$ ($n_s = 40000$)

1. *Metropolis Algorithm*
   i. Draw a candidate vector for the unbounded parameters (θ_*), from a multivariate normal distribution with mean θ_{s-1} and variance $\omega\mathbb{I}$, where ω is a scaling constant used to get an acceptance rate between 25% and 35%
   ii. Set

$$\theta_s = \begin{cases} \theta_* & \text{with probability } \eta \\ \theta_{s-1} & \text{with probability } 1-\eta \end{cases} \tag{3}$$

   for

$$\eta = min\left(1, \frac{p(y \mid f(\theta_*)^{-1})\, p(f(\theta_*)^{-1})\, J(\theta_*)}{p(y \mid f(\theta_{s-1})^{-1})\, p(f(\theta_{s-1})^{-1})\, J(\theta_{s-1})}\right) \tag{4}$$

2. Discard the first $s = 1, \dots, n_0$ ($n_0 = 20000$) draws of θ_s.

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Bayesian Estimation

Algorithm: Metropolis-Within-Gibbs

Recursion

1. *Metropolis Algorithm*
   Set Σ to the sample covariance of the chain of θ_s, ($s = \{n_0, \dots, n_s\}$), from the Initialisation step.
   For $q = 1, \dots, n_q$ ($n_q = 20000$)
   i. Draw a candidate vector for the parameters (θ_*), from a multivariate normal distribution with mean θ_{q-1} and variance $\omega\Sigma$, where ω is set to have an acceptance rate between 25% and 35%
   ii. Set
   $$\theta_q = \begin{cases} \theta_* & \text{with probability } \eta \\ \theta_{q-1} & \text{with probability } 1-\eta \end{cases} \tag{3}$$
   where η is defined as in the Initialisation step.
2. *Gibbs sampling*
   For $n_q > n_\emptyset$ for $n_\emptyset = 10000$ (burn-in period), apply the univariate approach for multivariate time series of of Koopman and Durbin (2000) to the simulation smoother proposed in Durbin and Koopman (2002) to sample the unobserved states, conditional on the parameters. In doing so, we follow the refinement proposed in Jarociński (2015).
3. Discard the first $q = 1, \dots, n_\emptyset$ draws of θ_q.

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