

ECON220B Discussion Section 4

Selection on Observables

Lapo Bini

UC San Diego

March 18, 2026

ROADMAP

1. Potential Outcomes and **SUTVA**
2. **Random Assignment**
3. **Unconfoundedness**
4. **IPW** and **RAE**
5. Exercise: What the ATE **misses**

INTRODUCTION TO POTENTIAL OUTCOMES

Binary treatment: an individual i could go into treatment or not. The **treatment status** is denoted by $D_i = \mathbb{1}\{\text{treated}\}$ and the **observed outcome** by Y_i .

► SUTVA

The observed outcome for unit i , Y_i , **depends only upon its own treatment** D_i , that is, the potential outcome is a function of the treatment status $Y_i := Y_i(D_i)$

► Potential Outcomes

For each i , we imagine there are two potential outcomes: outcome if **not treated** $Y_i(0)$, outcome if **treated** $Y_i(1)$.

► Observed Outcome Decomposition

Since treatment status is binary, we can decompose the observed outcome as

$$Y_i(D_i) = D_i Y_i(1) + (1 - D_i) Y_i(0)$$

DIFFERENCE IN MEANS?

- ▶ Our goal is to learn some statistical properties of $\tau_i = Y_i(1) - Y_i(0)$, for instance the **average treatment effect** $\tau_{ATE} := E[\tau_i]$
- ▶ Can we learn something useful from a **difference in means**? Let's look at plim:

$$\hat{\tau} = \left[\frac{1}{n_1} \sum_{i \in \mathcal{I}: D_i=1} Y_i \right] - \left[\frac{1}{n_0} \sum_{i \in \mathcal{I}: D_i=0} Y_i \right]$$

RANDOM ASSIGNMENT

► Selection Bias

In large sample, the **difference in means** converges to:

$$\hat{\tau} \xrightarrow{P} \underbrace{\left\{ E[Y_i(1) - Y_i(0) | D_i = 1] \right\}}_{\text{ATT}} + \underbrace{\left\{ E[Y_i(0) | D_i = 1] - E[Y_i(0) | D_i = 0] \right\}}_{\text{Selection Bias}}$$

► Random Assignment

A way to make progress is to exploit the (quasi) randomness of D_i : the treatment assignment D_i is **independent** of the potential outcomes $D_i \perp Y_i(1), Y_i(0)$.

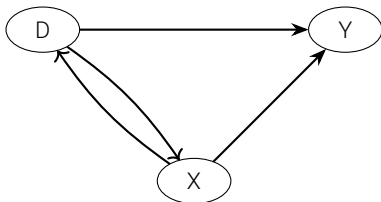
Very strong assumption, but it is a useful benchmark. **Implications:**

1. $\hat{\tau} \xrightarrow{P} \tau_{ATE}$
2. $\tau_{ATE} = \tau_{ATT} = \tau_{ATU}$
3. Selection bias = 0. Intuition?

UNCONFOUNDEDNESS

► Implausible Assumption

Random assignment is often **implausible** in economics research. Treatment take-up is generally non-random, it may depend on **covariates**, which also affect the realized outcome.



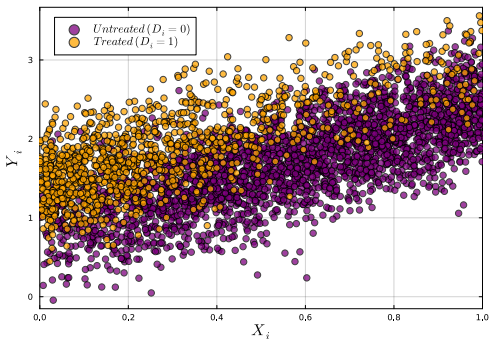
► Unconfoundedness

Also known as **selection on observables**, we assume that the treatment assignment D_i is **conditionally independent** of potential outcomes $D_i \perp (Y_i(1), Y_i(0)) \mid X_i$.

The treatment take-up D_i is as good as random within each covariate cell $\{X_i = x\}$

INVERSE PROBABILITY WEIGHTING (I/II)

Framework: each observation i is a worker in a firm; X_i denotes the percentage of monthly working hours spent programming, D_i indicates participation in a programming course, and Y_i represents post-training earnings.



- ▶ Is the **probability of participation** associated with different levels of X_i ?
- ▶ Is the outcome at very high values of X_i **informative** about the treatment effect?

INVERSE PROBABILITY WEIGHTING (II/II)

► Propensity Score

Treatment take-up depends on observed characteristics: we define the propensity score as the **conditional probability** of going into treatment given the covariates:

$$e(X_i) := E[D_i|X_i] = P(D_i|X_i)$$

► IPW Estimator

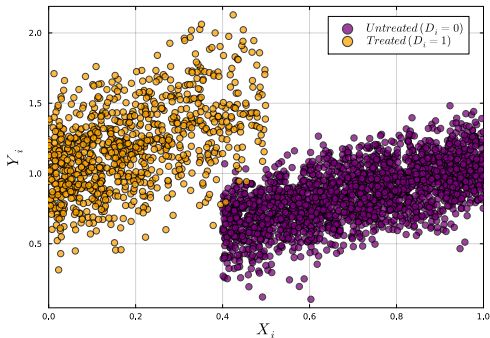
Can we learn something useful from a difference in means? **Yes**, if we use a **weighted average**!

From
$$\hat{\tau} = \left[\frac{1}{n_1} \sum_{i \in \mathcal{I}: D_i=1} Y_i \right] - \left[\frac{1}{n_0} \sum_{i \in \mathcal{I}: D_i=0} Y_i \right]$$

To
$$\hat{\tau}_{IPW} = \left[\frac{1}{n} \sum_{i \in \mathcal{I}: D_i=1} \frac{Y_i}{e(X_i)} \right] - \left[\frac{1}{n} \sum_{i \in \mathcal{I}: D_i=0} \frac{Y_i}{(1 - e(X_i))} \right]$$

REGRESSION ADJUSTMENT ESTIMATOR (I/II)

Framework: each observation i is a worker in a firm; X_i denotes the percentage of monthly working hours spent programming, D_i indicates participation in a programming course, and Y_i represents post-training earnings. Only workers with a **low level** of X_i chose to **enter the treatment**.



REGRESSION ADJUSTMENT ESTIMATOR (I/II)

► Linearity Assumption

We model the conditional expectation of observed outcome using **linear regression** and **interaction effects**:

$$E[Y_i|X_iD_i] := (1 - D_i) \cdot X_i\delta_0 + D_i \cdot X_i\delta_1$$

$$\tau_{ATE} = E[Y_i(1) - Y_i(0)] = E[Y_i(1)] - E[Y_i(0)]$$

$$\tau_{ATE} = E[E[Y_i(1)|X_i] - E[Y_i(0)|X_i]]$$

$$\tau_{ATE} = E[E[Y_i(1)|X_i, D_i = 1] - E[Y_i(0)|X_i, D_i = 0]]$$

$$\tau_{ATE} = E[E[Y_i|X_i, D_i = 1] - E[Y_i|X_i, D_i = 0]]$$

$$\tau_{ATE} = E[g_1(X_i) - g_0(X_i)] = E[X_i\delta_1 - X_i\delta_0]$$

$$\tau_{ATE} = \mu_X(\delta_1 - \delta_0)$$

EXERCISE: WHAT THE ATE MISSES

Consider the following **interventions**:

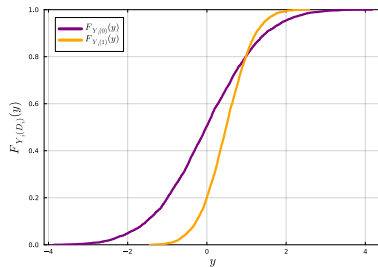
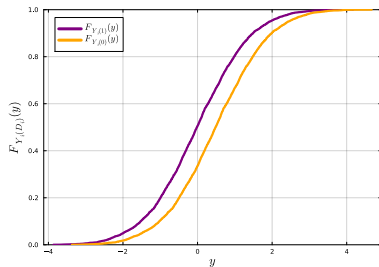
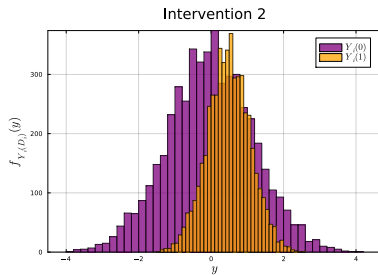
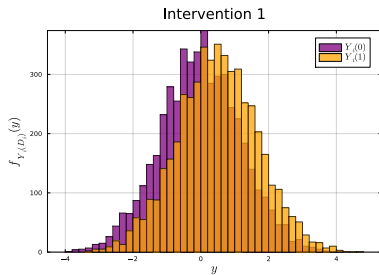
► **Intervention 1** $Y_i(1) = Y_i(0) + \delta \quad (\delta > 0) \quad \forall i.$

► **Intervention 2** $Y_i(1) = Y_i(0) + \delta \left\{ 1 + \frac{1}{2\delta} (E[Y_i(0)] - Y_i(0)) \right\} \quad (\delta > 0) \quad i.$

Questions (assume SUTVA, Unconfoundedness, Overlap)

- (1) Are the **average treatment effects** of the two interventions the same?
- (2) Which intervention is a **Pareto improvement**? What is the goal of the second intervention?

EXERCISE: WHAT THE ATE MISSES



EXERCISE: WHAT THE ATE MISSES

Consider the following **interventions**:

► **Intervention 1** $Y_i(1) = Y_i(0) + \delta \quad (\delta > 0) \quad \forall i.$

► **Intervention 2** $Y_i(1) = Y_i(0) + \delta \left\{ 1 + \frac{1}{2\delta} (E[Y_i(0)] - Y_i(0)) \right\} \quad (\delta > 0) \quad \forall i.$

Questions (assume SUTVA, Unconfoundedness, Overlap)

- (3) Prove that: $D_i Y_i = Y_i(1) D_i$ and $(1 - D_i) Y_i = (1 - D_i) Y_i(0)$. Given a generic function $h\cdot$, can we say that $D_i h\{Y_i\} = D_i h\{Y_i(1)\}$?
- (4) Let $h\{\cdot\}$ be any function, and recall $e(X_i) = P(D_i|X_i)$. Show that

$$E \left[\frac{h\{Y_i\} D_i}{e(X_i)} \right] = E[h\{Y_i(1)\}]$$

- (5) Claim: under an appropriate choice of $h\{\cdot\}$, the counterfactual distribution of the treated $F_{Y_1(d)}(y)$ is identified. Prove it.
- (6) Propose an estimator for $F_{Y_1(d)}(y)$ and $F_{Y_0(d)}(y)$.