

ECON220C Discussion Section 2

Fixed Effect Model

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ROADMAP

1. **Consistency**
2. **Identification Exercise**
3. **IVs and Clustering**

INTRODUCTION

► Fixed Effect Model

We assume that $\mathbf{x}_{it}$ and α_i are **correlated**: $\text{Cov}(\mathbf{x}_{it}, \alpha_i) \neq \mathbf{0}$.

The model we consider is $y_{it} = \mathbf{x}_{it}\boldsymbol{\beta} + \alpha_i + u_{it}$

► Remove Individual Fixed Effect

We want to get rid of α_i and obtain a transformed model such as $\tilde{y}_{it} = \tilde{\mathbf{x}}_{it}\boldsymbol{\beta} + \tilde{u}_{it}$, and then apply Pooled OLS. Different approaches:

1. **Within approach** $\tilde{y}_{it} \equiv y_{it} - \bar{y}_i$
2. **First difference** $\tilde{y}_{it} = \Delta y_{it} \equiv y_{it} - y_{it-1}$
3. **Forward demeaning** $\tilde{y}_{it} \equiv y_{it} - \bar{y}_i$ (or backward)

CONSISTENCY & UNBIASEDNESS

► Framework

Consider the model $y_{it} = x_{it}\beta + \alpha_i + u_{it}$. To eliminate the unobserved individual effect α_i and consistently estimate the slope vector β , we apply two specific transformations to the data, $\tilde{x}_{it}$ and $\tilde{y}_{it}$, and then apply the pooled OLS formula:

$$\hat{\beta} = \left(\sum_{i=1}^N \sum_{t=1}^T \tilde{x}_{it}' \tilde{x}_{it} \right)^{-1} \left(\sum_{i=1}^N \sum_{t=1}^T \tilde{x}_{it}' \tilde{y}_{it} \right)$$

► Questions

- (i) Under what conditions is $\hat{\beta}$ **consistent** for β for a fixed T as $N \rightarrow \infty$ if we use the first difference or within approach?
- (ii) Under what conditions is $\hat{\beta}$ **conditionally unbiased** for β if we use the first difference or within approach?

IDENTIFICATION

► Framework

Consider the **model** $y_{it} = x_{it}\beta + z_i\gamma + \alpha_i + u_{it}$ where $u_i \mid \mathbf{X}_i \mathbf{Z}_i \sim \mathcal{N}(\mathbf{0}, \sigma_u^2 \mathbb{I}_T)$

Suppose u_i is independent of α_i , and x_{it} is a scalar.

► Questions

- (i) If $\alpha_i \mid \mathbf{X}_i \mathbf{Z}_i \sim \mathcal{N}(0, \sigma_\alpha^2)$, are β and γ **identified**? Do we need any extra **assumptions**?
- (ii) If α_i is **correlated** with x_{it} and z_i , are β and γ identified?
- (iii) Suppose now to have $x_{it} = (x_{1it} \ x_{2it})$ and $z_i = (z_{1i} \ z_{2i})$, where:
 - x_{1it} is correlated with z_{1i} and α_i ,
 - x_{2it} is uncorrelated with α_i but correlated with z_{1i} ,
 - and z_{2i} is uncorrelated with α_i . Are β and γ **identified**?

IV & CLUSTERING

► Framework

Consider the **linear model** $y_{it} = x_{it}\beta + \alpha_i + u_{it}$ where x_{it} may be correlated with α_i and $u_{is} \forall s$. There is an instrumental variable z_{it} that is correlated with α_i but $E[u_{it}|z_{it}] = 0$ for all i and t . Consider the following estimator:

$$\hat{\beta} = \frac{\sum_i \sum_t (z_{it} - \bar{z}_i) (y_{it} - \bar{y}_i)}{\sum_i \sum_t (z_{it} - \bar{z}_i) (x_{it} - \bar{x}_i)}$$

► Questions

- (i) Under what conditions is $\hat{\beta}$ consistent?
- (ii) Is the condition $E[u_{it}|z_{it}] = 0 \forall i, \forall t$ enough for consistency?
- (iii) Is the condition $E[u_{it}|z_{is}] = 0 \forall i, \forall t, \forall s = 1, \dots, t$, enough for consistency?
- (iv) Now suppose $\hat{\beta}$ is consistent, and we want to compute its standard error. Explain when you need to use the cluster-robust standard error.
- (v) Explain how you would estimate the asymptotic variance of your estimator.