

ECON220C Discussion Section 3

Clustering & Time Fixed Effect

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ROADMAP

1. **Clustering**

2. **Time Fixed Effect**

CLUSTERING

► Framework

Consider the **linear model** $y_{it} = x_{it}\beta + \alpha_i + u_{it}$. Clustering means asking ourselves: what is a realistic **covariance structure** across individuals and over time?

$$\hat{\beta} = \left(\sum_i \sum_t \tilde{x}_{it}' \tilde{x}_{it} \right)^{-1} \left(\sum_i \sum_t \tilde{x}_{it}' \tilde{y}_{it} \right)$$

TIME FIXED EFFECT

► New Source of Variation

Consider a panel data model with **person specific** and **time fixed effects**, where we observe each individual for multiple periods: $y_{it} = x_{it}\beta + \gamma_t + \alpha_i + u_{it}$.

► Interpretation

Here α_i is an unobserved person specific shock, γ_t is an unobserved time specific shock, and u_{it} an unobserved idiosyncratic shock.

► Underlying Assumptions

Under what conditions is the model above realistic?

EXERCISE

► Framework

Consider the following **model**: $y_{it} = x_{it}\beta + \gamma_t + \alpha_i + u_{it}$. Assume $\{y_{it}, x_{it}, \alpha_i, \gamma_t, u_{it}\}_{i,t}$ iid, U_i independent of $(X_i, \gamma_t, \alpha_i)$, $\text{Cov}(\alpha_i, x_{it}) \neq 0$, and $E[u_{it}] = E[\alpha_i] = 0$. All the random variables have finite second moments.

► Questions

- (i) Suppose $T = 2$, are **first difference** and the **within approach equivalent**?
- (ii) Suppose $T = 2$, use the first difference approach and show that β is **identified** under the condition $E[\Delta x'_{it}(\Delta x_{it} - E[\Delta x_{it}])]$ full rank.
- (iii) Consider running an OLS regression with a constant on the first difference, i.e. $\tilde{x}_{it} = (1 \ \Delta x_{it})$. Let:

$$\left(\hat{\kappa} \ \hat{\beta} \right) = \arg \min_{(k,b)} \frac{1}{n} \sum_{i=1}^n (\Delta y_{it} - k - \Delta x_{it}b)^2$$

characterize the **joint asymptotic distribution** of the two estimators.

- (iv) Propose two estimators for γ_1 and γ_2 .
- (v) Suppose $T = 3$, under what conditions is **first difference + Pooled OLS consistent**?
- (vi) Suppose $T = 3$, propose a consistent estimator for β .